Inverse Problem Seminar from Theory to Applications –Finland-Japan Seminar for Inverse Problem–

On 27th of January 2025 and 28th, the Department of Mathematics of Josai University is proud to host “ Inverse Problem Seminar from Theory to Applications ” at Tokyo Kioi-cho Campus of Josai University, Tokyo, Japan. The seminar aims to maintain and enhance collaboration among mathematicians of Finland and Japan. This seminar create a platform for scientific discussions in areas of Inverse Problems from the theory to real world applications. Seminar proceedings will be published in Josai Mathematical Monographs (pier review). The deadline of submission for seminar proceeding is 30th of April, 2025.

Scientific organizer : Takanori Ide (Department of Mathematics, Faculty of Science, Josai University)

Schedule Day 1

27th of January (Mon), 2025

Chairperson Takanori Ide

10:00–11:30 (Keynote speech) Masaru Ikehata (Professor Emeritus at Hiroshima University, Professor Emeritus at Gunma University)
Title : Analytical methods for inverse obstacle problems

11:30–13:00 Lunch

Chairperson Samuli Siltanen

13:00-13:45 Hiroshi Isozaki (Ritsumeikan University)
Title : On the uniqueness of scattering solutions to partial differential equations

13:45-14:30 Gen Nakamura (Dept. Math. and Research Institute of Electronic Science, Hokkaido University)
Title : Analysis on anisotropic spring-dashpot models and Boltzmann type viscoelastic systems

14:30-15:00 Coffee Break

Chairperson Hisashi Morioka

15:00-15:45 Mishio Kawashita(Hiroshima University)
Title : On active sonar-type wave scattering inverse problems for different types of cavities

15:45-16:30 Daisuke Kawagoe(Kyoto University)
Title : Discontinuities in stationary radiative transfer

and the optical tomography

16:30-17:15 Akari Ishida(Nagoya University)
Title : Convergence analysis of the Levenberg-Marquardt method for inverse problems satisfying Hölder stability

Schedule Day 2

28th of January (Tue), 2025

Chairperson Hiromichi Itou

10:00-11:30 (Keynote speech) Samuli Siltanen (University of Helsinki)
Title : Tomographic imaging: probing with X-rays or electricity

11:30-13:00 Lunch

Chairperson Mishio Kawashita

13:00-13:45 Siiri Rautio (University of Helsinki)
Title : Computed tomography without X-rays: parallel-beam imaging from nonlinear current flows

13:45-14:30 Kazumi Tanuma (Gunma University)
Title : Surface waves in piezoelectric media: surface-impedance approach

14:30-15:00 Coffee Break

Chairperson Daisuke Kawagoe

15:00-15:45 Hisashi Morioka(Ehime University)
Title : Scattering theory for elastic wave equations in cylindrical domains with thin bond layers

15:45-16:30 Ryusei Yamashita(Polytechnic University of Japan)
Title : Reconstruction of the defect by the enclosure

method for inverse problems of the magnetic Schrödinger operator

Keynote speech : Analytical methods for inverse obstacle problems

Masaru Ikehata[1 2]

[1]Professor Emeritus at Hiroshima University
[2]Professor Emeritus at Gunma University

This talk is concerned with methodology on the reconstruction issue for inverse obstacle problems governed by partial differential equations and consists of three sections. First an introduction of representative classical analytical methods, which are the linear sampling and factorization methods, the probe and singular sources methods and the enclosure method is given. Second the recent development of the enclosure method in time domain is described. It is focused on showing three types of applications of the method to inverse obstacle problems governed by wave equations. Finally, as a most recent topic, an integrated theory of the probe and singular sources methods is presented.

On the uniqueness of scattering solutions to partial differential equations

Hiroshi Isozaki[1]

[1]Ritsumeikan University

In the study of scattering of waves, a crucial role is played by the Sommerfeld radiation condition and the Rellich type uniqueness theorem. The former singles out the asymptotic form of scattering solutions to the Helmholz type equation, and the latter determines its sharp decay rate. They ensure the non-existence of eigenvalues embedded in the continuous spectrum, what is more, they turn out to be a key step toward the inverse scattering. We will discuss these problems in two topics. The first issue, a joint work with Matti Lassas, deals with a class of Riemannian manifolds, for which we solve the inverse scattering problem. The second issue, a joint work with Mitsuteru Kadowaki and Michiyuki Watanabe, is concerned with the elastic equation in a perturbed 3-dim. half space, for which we introduce the radiation condition and prove the uniqueness of scatteirng solutions.

Analysis on anisotropic spring-dashpot models and Boltzmann type viscoelastic systems

Gen Nakamura[1]

[1] Dept. Math. and Research Institute of Electronic Science, Hokkaido University

Spring-dashpot models (abbreviated by SDM) and Boltzmann-type viscoelastic system of equations (abbreviated by BVS) are two major types of systems of equations for viscoelasticity. The convolution kernel of the stress-strain relation for the SDM is called the relaxation tensor. If the relaxation tensor is derived, the SDM can be converted to the BVS. Concerning the relaxation tensor, the typical SDM are the Maxwell model, Burgers model, standard linear solid model, and their extended versions. By measuring the relaxation time, we can have qualitative information on the relaxation tensor.

The derivation of the relaxation tensor for the anisotropic extended Burgers model has not been known. In the first part of my talk, I will show how to derive the relaxation tensor for this model and analyze its properties (see [4]). Further, I will show the exponential decay of solutions of the initial boundary value problem (abbreviated by IBP) with the mixed-type boundary condition for the BVS (see [2]).

The second part of my talk is about the control theory for the BVS via SDM. Since the BVS is an integrodifferential system of equations, solutions of the IBP with mixed-type boundary condition for the BVS do not generate semigroup. This is one of the major reasons that it is hard to study the control theory for the BVS. While, for those of the extended Maxwell model, I will show in the second part of my talk that we can have the generation of group (see [3]). However, it has a stationary solution. To avoid this, we consider its subsystem which we call the reduced system. Solutions of its IBP with mixed-type boundary condition decay exponentially in time. Based on this, I will discuss the exact source controllability for the BVS associated to the extended Maxwell model.

If there is some time left, I will touch on an inverse problem for the BVS as the continuation of my study giving the foundation for the vibroseis reflection exploration of anisotropic viscoelastic grounds when their physical properties are either piecewise homogeneous or piecewise isotropic (see [1], [5], [6]).

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On active sonar-type wave scattering inverse problems for different types of cavities

Mishio Kawashita[1]

[1] Hiroshima University

This talk presents a mathematical analysis of the inverse scattering problem, which is formulated as an active sonar-type scattering problem. Obtaining information about an object through observation of reflected waves from an incident wave emitted from "a certain location" is an important problem in applications such as ocean exploration and breast cancer screening, and has been actively studied. In this talk, we refer to this form of problem as an active sonar-type scattering problem. To limit the discussion, we assume that the wave propagation is governed by the wave equation and that the only object D for which we want to know the information is cavities. An incident wave is launched from $B \subset \mathbb{R}^3$ at time 0 and is observed at the same location B until time $T > 0$. The observed data are given by the solution of the wave equation at $[0, T] \times B$. The objective is to gain insight into cavity D based on the observed data.

An effective mathematical analysis method for active sonar-type wave-scattering inverse problems is the enclosure method. The enclosure method was developed by Masaru Ikehata for the cavity estimation problem of the Laplace equation. It has also been confirmed to be effective for time-dependent systems such as wave and heat equations. In the enclosure method, similar to other mathematical methods for solving inverse problems, a function called the indicator function is introduced based on the observed data, and information about the object is obtained through its analysis. It is known that the indicator function I_τ in the enclosure method contains a large parameter $\tau > 1$, and the distance $\mathrm{dist}(D, B)$ between object D and observation location B is obtained from the asymptotic behavior of I_τ when $\tau \to \infty$. The enclosure method is valid even when D consists of multiple cavities, and much is already known about this. However, previous studies on cavity estimation have only considered cases in which the indicator functions have the same sign. For example, $I_\tau < 0$ for cavities with Dirichlet boundary conditions, but $I_\tau > 0$ for cavities with Neumann conditions. Therefore, it is questionable what happens when these cavities are colocated. In this talk, according to [1] and [2], we present what happens when different types of cavities with different signs of the indicator function are mixed.
This is a joint work with Wakako Kawashita of Hiroshima University.

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Discontinuities in stationary radiative transfer and the optical tomography

Daisuke Kawagoe[1]

[1] Kyoto University

The optical tomography is a new medical imaging technology using near infra-red light, and it is mathematically modeled as an inverse problem to determine the attenuation coefficient in the stationary (radiative) transport equation (STE) from boundary measurements. We discussed discontinuity in the solution to the (forward) boundary value problem arising from discontinuous incoming boundary data, which we call the boundary-induced discontinuity. In particular, assuming a condition on the set of discontinuous points of the coefficients, we gave two kinds of the incoming boundary data for the boundary-induced discontinuity in two or three dimensional bounded convex domains, and described how it propagates in those situations. From properties of the boundary-induced discontinuity, we proposed a method to solve the aforementioned inverse problem. In this talk, we remove the condition on the discontinuous points of the coefficients and also discuss the coefficient-induced discontinuity, discontinuity in the solution to the boundary value problem arising from discontinuity of the coefficients.

Convergence analysis of the Levenberg-Marquardt method for inverse problems satisfying Hölder stability

Akari Ishida[1]

[1] Nagoya University

We analyze the convergence of the nonlinear Levenberg-Marquardt method for inverse problems in Hilbert spaces. We establish convergence and convergence rates for a class of inverse problems that satisfy Hölder stability. Furthermore, grounded in our established results, we develop reconstruction algorithms for solving inverse problems with finite measurements for exact and noisy data, respectively. This work is based on joint research with Sei Nagayasu (University of Hyogo) and Gen Nakamura (Hokkaido University).

Keynote speech : Tomographic imaging: probing with X-rays or electricity

Siltanen Samuli[1]

[1] University of Helsinki

In the 1970 ’ s, a new X-ray based innovation was introduced. Tomography, or slice imaging, revealed the inner structure of a patient point by point as a three-dimensional map of tissues. This opened up a new world for doctors as they could do precise diagnosing based on these "CAT-scans." Tomography is based on recording X-ray images of the patient along many directions, and then using mathematics in a clever way for combining the information into a 3D image. This talk explains that process in simple terms. An important research topic in modern mathematics is to look for a way to do tomographic imaging with the least possible amount of radiation dose to the patient. Or sometimes to compensate for incomplete measurements caused by restrictions in the imaging arrangement. This is based on a process called regularisation, also illustrated in the talk in an easy-to-understand way. Also: there is a fun quiz revealing natural tomographers among the audience. In electrical impedance tomography (EIT), one feeds electric currents into a physical body through electrodes attached to the surface of the body. The resulting voltages at the electrodes are measured. From this data one attempts to recover the electric conductivity distribution inside the body as an image. The inverse problem of EIT is nonlinear and ill-posed, so the design of reconstruction algorithms is challenging. Presented is an approach called the D-bar method, based on a nonlinear low-pass filter.

Computed tomography without X-rays: parallel-beam imaging from nonlinear current flows

Siiri Rautio[1]

[1] University of Helsinki

We introduce a new reconstruction algorithm for electrical impedance tomography (EIT), that provides a curious connection between EIT and traditional X-ray computed tomography, based on the idea of "virtual X-rays". We show that the exponentially ill-posed and nonlinear inverse problem of EIT can be divided into separate steps. We start by mathematically calculating so-called virtual X-ray projection data from the measurement data. Then, we perform explicit algebraic operations and one-dimensional integration, ending up with a blurry and nonlinearly transformed Radon sinogram. Then, we use a neural network to remove the higher-order scattering terms and perform deconvolution. Finally, we can compute a reconstruction of the conductivity using the inverse Radon transform. We demonstrate the method with experimental data.

Surface waves in piezoelectric media: surface-impedance approach

Kazumi Tanuma[1]

[1] Gunma University

In piezoelectricity, the mechanical stress and the electric displacement are related to the mechanical displacement and the electric potential through the elasticity tensor, the piezoelectric tensor and the dielectric tensor. It is known that surface waves called Bleustein-Gulyaev (BG) waves propagate along the surface of the 3D Euclidean half-space occupied by a homogeneous transversely-isotropic piezoelectric medium. The surface is subject to the mechanically-free and electrically-closed (i.e., grounded) condition. We consider a time-harmonic solution which pertains to subsonic BG waves and define the surface impedance matrix which expresses a linear relationship between (i) the mechanical displacement and the normal component of the electric displacement at the surface on which BG waves propagate and (ii) the mechanical traction and the electric potential needed to sustain them at that surface. The phase velocity of BG waves makes the surface impedance matrix have a zero eigenvalue, whereas the polarization vector of BG waves at the surface generates the null-space of the matrix. Suppose that a fully anisotropic perturbation is added to the transversely-isotropic material constants of the piezoelectric half-space. We are then interested in observing how BG waves deviate from their original state of shear-horizontal modes. We investigate the perturbations of their phase velocity and their polarization vector at the surface as well as the stability of those perturbed waves.

Scattering theory for elastic wave equations in cylindrical domains with thin bond layers

Hisashi Morioka[1]

[1] Ehime University

We consider the scattering for the elastic wave equation in a cylindrical stratified media via incident waves vertical to the interfaces. Then the problem can be reduce to one-dimensional wave equations. This study is inspired by a non-destructive testing via ultrasonic waves for medium with some thin bond layers. We also consider an inverse problem in view of the bond correction method and the pressure dependence of this method.

Reconstruction of the defect by the enclosure method for inverse problems of the magnetic Schrödinger operator

Ryusei Yamashita[1]

[1] Polytechnic University of Japan

This study is based on the paper [4] and [5]. We show a reconstruction formula of the convex hull of the defect D from the Dirichlet to Neumann map associated with the magnetic Schrödinger operator

$$D_A^2 u + qu = 0 \quad in \quad \Omega \backslash \bar{D} \quad (\bar{D} \subset \Omega)$$

under the Dirichlet condition or the Robin condition on the boundary ∂D in the two and three dimesional case. Here, for $A = (A_1, A_2, \cdots, A_n)$, let

$$D_A^2 := \sum_{j=1}^{n} D_{A,j}(D_{A,j}u), \quad D_{A,j} := \frac{1}{i}\partial_j + A_j.$$

We use the enclosure method proposed by Ikehata[1]. Ikehata gave the formula to reconstruct the defect D for the Helmholtz equation under the Robin condition on ∂D in [2]. We extended this result for the magnetic Schrödinger operator under the Robin and Dirichlet condition on ∂D in the two and three dimensional case in [4]. Recently in [3], Ikehata established the new estimates of the stationary Schrödinger equation without magnetic field. So, we extend this estimate for the magnetic Schrödinger operator under the Robin condition on the boundary D in [5].

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